Rice, L. (2015). Prospective secondary mathematics teachers' thinking in arguing: Applying Toulmin's scheme to four cases. *The Researcher, 27*(2), 20-24.

Prospective Secondary Mathematics Teachers' Thinking in Arguing: Applying Toulmin's Scheme to Four Cases

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Abstract: This study examined four prospective mathematics teachers thinking while creating and analyzing mathematical arguments. The data was examined through Toulmin's (1958) argumentation scheme to understand how participants created their arguments. Findings suggest several areas in which they could strengthen their arguments.
Keywords: Arguing, mathematics, prospective teachers

The importance of engaging students in arguing to learn mathematics is prevalent in the literature (Forman, Larreamendy-Joerns, Stein, & Brown, 1998; Walshaw & Anthony, 2008). Two influential organizations emphasize the importance of engaging students in arguing in the mathematics classroom: National Council of Teachers of Mathematics (NCTM, 2000) and the National Governors Association Center for Best Practices & Council of Chief State School Officers (NGA & CCSSO, 2010). It is important for students to make mathematical arguments in classrooms because it facilitates learning. Thus, it is a valuable teaching practice to engage students in arguing. Arguing has been called "the ideal mathematical inquiry" (Kennedy, 2009, p. 73). Students who engage in arguing exchange mathematical ideas, which can improve their understanding of those ideas. A classroom that supports argumentation can encourage students to make conjectures, discuss their ideas, and make conclusions.

Researchers have voiced their concerns of whether teachers are capable of teaching proof and argumentation (e.g., Reid & Knipping, 2010). This raises questions of how well-prepared prospective teachers are to engage their students in argumentation. Morris (2007) found that prospective teachers struggle to evaluate arguments and held misconceptions about arguing. The purpose of this study was to examine prospective secondary mathematics teachers' thinking in arguing in mathematics.

RESEARCH METHODOLOGY

Four prospective secondary mathematics teachers participated in this study during spring and summer of 2013. At that time they were student teaching at junior highs or high schools. The research question was: What are characteristics of prospective mathematics teachers arguments? To gather and analyze data, two

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methods were used: multiple case study (Stake, 2006) and Toulmin's (1958) argumentation scheme. Seven task-based interviews were conducted with participants. Each interview was recorded and transcribed. Participants were presented with mathematical tasks, asked to analyze the argument embedded in the tasks or asked to create an argument to address the task. The mathematical tasks included mathematical ideas from high school and college courses. For example, mathematical ideas found in calculus and number theory courses inspired some tasks.

To uncover the different characteristics of participants' arguments, Toulmin's (1958) argumentation scheme was the tool of analysis. Though not developed for analyzing arguments in mathematics, researchers have adopted his scheme to study mathematical argumentation (e.g., Inglis, Mejia-Ramos, & Simpson, 2007; Weber, Maher, Powell, Lee, 2008). Toulmin's (1958) scheme is a tool to deconstruct arguments. Elements of an argument are categorized as either a *conclusion*, which some refer to as *claim* (Weber et al., 2008), *data*, *warrant*, *backing, modal qualifier*, or *rebuttal*. The conclusion (or claim) is the statement one wants to establish as true. Data is the evidence for the conclusion and builds foundation for the argument. A warrant connects the data and conclusion together, which can be done by "appealing to a rule, a definition or by making an analogy" (Inglis et al., 2007, p. 4). A backing is supplemental evidence. A modal qualifier "qualifies the conclusion by expressing degrees of confidents" (Inglis et al., 2007, p. 4). A rebuttal is a possible refutation to the conclusion and indicates when the conclusion would fail to be true. An example of a task presented to participants was: *For all natural numbers n,* $n^3 - n$ *is divisible by 3. Is this true?* The participant was then asked questions about whether or not she thought it was true, how she would argue the truth or falsity of it, and then asked to create an argument to conclude if it was true or false.

RESULTS

Analyzing the participants' responses in terms of Toulmin's (1958) argumentation scheme revealed aspects about their ways of constructing arguments. Looking across cases for similarities and differences provided insight into individual participant's arguing and ways of arguing common among the participants. Participants primarily initiated empirical arguments to create an argument, unless they recalled a proof they had seen before. In the latter case, they tried to recreate the proof. The data looked similar across tasks, which typically consisted of cases appropriate to the situation. The participants provided data to support their argument of the truth of a claim. Using cases was enough to convince participants of the truth of a claim. There was also evidence that some participants could critique and use warrants to connect the data and conclusion together. All four participants considered rebuttals in at least one task when either analyzing a given argument or when constructing an argument. Though their discussion of possible refutations was a small part of the argument they created, it shows that participants have the potential to create complex arguments. Two ways this took the form was predicting student opposition to the claim and verifying cases to see if they could generate a counter example. In general, however, the arguments they created were based on a small number of cases, which would not be accepted in advanced mathematics courses.

It is important to mention there was no evidence to suggest that participants gave more supporting evidence to any warrants they provided (backings) nor did they address the issue of under what conditions did the argument hold (modal qualifiers). Warrants are important because they "remove uncertainty" from arguments (Inglis et al., 2007, p. 15). However, warrants need to be used in conjunction with modal qualifiers to effectively remove that uncertainty (Inglis et al., 2007). In light of this, the participants' arguments could be strengthened by the inclusion of modal qualifiers. Overall, the findings suggest that participants could create arguments based on cases, but in terms of Toulmin's (1958) scheme the arguments were incomplete. It is not possible to generalize results because of the small sample size. These findings are summarized in Table 1.

Table 1. *Summary of participants' data related to Toulmin's (1958) framework*

Pseudonym	Summary
Nicole	May consider rebuttals, but only one recorded incident: "You check and see" to see if claim holds. Provided data to support arguments: cases, specific values. Made conclusions.
Cooper	Struggled with backings. Minimally considered rebuttals. One instance of using warrant: numerical reasoning to explain there will be one quantity in $(n-1)(n)(n+1)$ that is a multiple of 3 would be considered a warrant. Provided data to support arguments: tested cases.
Jenny	Used and critiqued warrants. Considered rebuttals: predicted students would try to find counter examples. Provided data to support arguments.
Becky	Could critique warrants. Struggled to use them to build arguments. Provided data to support arguments. Considered rebuttals.

IMPORTANCE TO THE FIELD

The differences in participants' responses provided insight into their dispositions towards mathematics. There were instances when a participant gave up on a task quickly or refused to attempt a problem, while others worked on tasks tirelessly or outside of interviews. The latter are qualities we want teachers to instill in students. The former are qualities that contradict the Common Core standard for mathematical practice "Make sense of problems and persevere in solving them" (NGA & CCSSO, 2010, p. 6). When a participant quickly gave up on a task, though not every participant did, it begged the question of how they would react to a student who refuses to try a problem. It may be beneficial to look at certain aspects of their dispositions, such as perseverance and risk-taking in mathematics.

The findings suggest these pre-service teachers could construct arguments with some elements of Toulmin's (1958) scheme. However, there were qualities missing from their arguments that would make them stronger. Based on this finding, two implications emerge concerning the participants' content knowledge and pedagogical content knowledge. These pre-service teachers omitted certain elements from their arguments, rendering them incomplete. A reason why may be they do not understand what makes a rigorous argument, or they may not know how to create one. From a content knowledge perspective, creating rigorous arguments is valuable for doing mathematics. It can reflect one's depth of understanding of a concept. Their incomplete arguments may imply they have an area of weakness in mathematics. Interview questions mainly focused on the creation and analysis of arguments; however, they were also posed questions about how to engage students in arguing in a mathematics classroom or to analyze a hypothetical classroom scenario.

Though participants fared better when analyzing arguments over creating arguments, there was still room to grow in analyzing arguments. One concern raised by the findings is how they would fare in the moment with students when they are mathematics teachers. It seems likely they would struggle to create rigorous arguments for and analyze claims and arguments given by students. This concern may interest those who involved in pre-service mathematics teacher preparation. A possible direction to turn is investigating where and how arguing is addressed in mathematics teacher preparation programs.

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