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How Do Teacher Candidates Perceive the Concept of Zero?

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Abstract: *This study was to determine how teacher candidates perceive zero. In this study, which used the case study method from qualitative research approaches, 264 teacher candidates constituted the participants. The participants consisted of 66 elementary mathematics, 58 special education, 82 primary school and 58 preschool teacher candidates. The questions in data collection tool that do not require knowledge at the level of expertise were created. The research was conducted with candidate teachers studying in different departments. It was found that there were no obvious differences between the candidates' perceptions of zero.*

Key Words: Zero concept, Zero perception, Nature of zero, Candidate teacher

INTRODUCTION

The numeral zero invented by the Indians (Kaplan, 1999; Seife, 2000) played an important role in the development of the decimal number system (Anthony & Walshaw, 2004; Hughes, 1986). Although zero is important in the number system, it was discovered much later than other numerals (Altoğ, 2016; Lewis, 2015). Even, until the 1800s, zero was not accepted as a symbol (Anthony & Walshaw, 2004). The late entry and acceptance of mathematics indicates that zero is a special number. Because of zero's characteristics, it also is a concept that must be treated with care (Altoğ, 20106; Cankoy, 2010). For example, there is no common idea whether zero is a natural number or not (Erdoğan, 2019). At the undergraduate level, zero is not included in the set of natural numbers due to the Peano axioms (Halilov *et al.,* 2008). When the textbooks at other levels are examined, zero is accepted as a natural number. Another situation is that although the zero symbol was not accepted until the 1800s, it eliminated the requirement to meet the expression of absence in the number system (Anthony & Walshaw, 2004). Regardless of the time, zero has been an incomprehensible concept for learners, probably due to its different meanings (Wheeler & Feghali, 1983).

So, why has "zero" been a difficult concept? Mathematicians use zero in an algorithm or to count an empty set, while other individuals express zero as an absence (Anthony & Walshaw, 2004; Kaplan, 1999). So it can be thought that zero is expressed differently for mathematicians and others, especially students. Zero is not only associated with "absence" as mentioned, but also with concepts such as "neutral", "starting point" (Cockburn & Parslow-Williams, 2008, as cited in Bütün & Erdoğan, 2020; Cüceloğlu, 2014; Erdoğan, 2019). Many similar properties can be said.

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in which zero is identified. So zero with different meanings is difficult to receive. In the studies conducted, it is mentioned that besides understanding zero, it is also difficult to teach it (Ma, 2010; Tsamir *et al.,* 2000). Loading the meaning of absence to zero makes it difficult for young students to perceive zero (Haylock & Cockburn, 2013/2014).

Students encounter numbers even before they start their school life. They deal with numbers in their daily life and throughout their learning lives (Erdoğan, 2019). Even counting and teaching of one-digit numbers, that is, numerals, are included in the curricula of the preschool period (CCSSI, 2010). Considering the fact that prior learning will form the basis for later learning, the concept of numerals and numbers learned at an early age will have an effect on students' mathematics success in advanced grades (Haylock & Cockburn, 2013/2014; Locuniak & Jordan 2008; Jordan *et al.,* 2010). For example, if the numbers are only interested in counting, there may be differences in the attitude of students towards negative numbers that they will learn in their later learning lives (Haylock & Cockburn, 2013/2014). Considering that the understanding of even negative numbers by younger students is also emphasized by the National Mathematics Teachers Council, it is important to make a conceptual and procedural meaning of zero (NCTM, 2000). It has been determined that students who are younger (4-8 years) use zero to refer to the number of elements of a set that does not have an element, or to the end or beginning of something. These students evaluated zero as a steady state or reference point on the number line. Apart from these, all students expressed zero as "absence" (Manchester, 2011). Studies on the concept of zero continued not only with pre-school students, but also with students in formal education, pre-service teachers and even teachers.

Anthony and Walshaw (2004) listed the problems related to the concept of zero. Some of these problems are the incomprehensibility of zero being a placeholder, dividing by zero, subtracting by zero, and signifying nonexistence by zero. It has been determined by the researches that both students, teacher candidates and teachers have similar problems. For example, it has been found that elementary school students have difficulty evaluating zero as an integer or rational number (Altoğ, 2016; Avcu & Durmaz, 2011; İşgüden, 2008; Temel & Eroğlu, 2014). It was determined by seven, eight, and ninth graders that the division of a number to zero could not be understood, and that the undefined division could not be expressed (Altoğ, 2016; Alkan, 2009; Özdeş, 2013). Cankoy (2010), who conducted his work with high school teachers, asked teachers what $a \div 0$ is and how to tell their high school students about it. It was determined that teachers, beyond teaching, could not explain what this section was. In addition, it is one of the results of some studies that it cannot be decided whether zero is odd or even. Altoğ (2016) found that eighth graders accepted the beginning of even numbers as two, so they could not think of the number zero as an even number. Levenson *et al.* (2007) have revealed that sixth graders think of zero as neither odd nor even number. In her study of primary school teacher candidates, Toluk Uçar (2011) stated that almost half of teacher candidates could not determine zero as an even number.

In similar studies conducted with students, teacher candidates and teachers, the mistakes and misconceptions about the concept of zero as a focus were investigated. Bütün and Erdoğan (2020) stated that most of the studies on the concept of zero reveal the lack of mathematical knowledge of teachers / candidate teachers regarding this concept, but this is not sufficient. When the literature is examined, it is seen that there are studies with teacher candidates and teachers that contain instructional explanations of zero (Bütün & Erdoğan, 2020; Cankoy, 2010; Erdoğan, 2019; Karakuş, 2017). Along with the instructional understanding, it is thought that it is necessary to examine how teachers perceive the concept of zero. Öçal and Kızıltaş's (2019) study includes this idea. In this study, Öçal and Kızıltaş (2019) investigated preschool teachers' perceptions of the

concept of zero and their teaching practices. In this study, which was carried out with eighteen preschool teachers, the teachers expressed zero as a symbol and absence. It is stated that teachers use zero in daily life in a way that includes these two situations, and they teach in line with these perceptions.

It is important to know the perception, thoughts and understanding of the concept of zero of teachers/candidate teachers who will teach zero beyond the mathematics content or teaching knowledge. The concept of zero is used not only in preschool but also in primary, elementary and special education mathematics. It is believed that it is necessary for candidates of teachers who will teach at these levels to know the concept of zero, the place of this concept in mathematics and the ways it is used in daily life to be able to teach zero. In addition, a study like this study (Wheeler & Feghali, 1983) was conducted only with primary school teacher candidates. A study conducted with candidate teachers from different department who will teach zero was not found in the literature. To help fill the gap in this field, in this study, "*How do teacher candidates perceive the concept of zero?*" the answer to the question was sought.

METHOD

RESEARCH DESIGN

In this study, it was tried to determine how teacher candidates perceive the concept of zero. For this purpose, the case study method was adopted from qualitative research approaches. Besides the search for answers to the question of how, the case study gives the opportunity to examine a phenomenon in depth other than the intervention of the researcher (Yıldırım & Şimşek, 2013). In the case study method, the data obtained from the participants are detailed and presented in case themes (Creswell, 2013/2015). In this study, in accordance with the explanations of Yıldırım and Şimşek (2013) and Creswell (2013/2015), the perceptions of teacher candidates towards the concept of zero were examined in detail. In addition, the data obtained from teacher candidates are divided into status themes and presented in depth.

PARTICIPANTS

The participants consist of 264 teacher candidates studying in the third grade of elementary mathematics, special education, primary school and preschool teaching at a state university. Firstly, candidate teachers were selected with the appropriate sampling method. In order to collect data faster, the researchers preferred a faculty of education belonging to a university that they could reach more easily. Afterwards, participants were formed by applying the purposeful sampling method in the determined education faculty. The participants consisted of 66 elementary mathematics, 58 special education, 82 primary school and 58 preschool teaching third grade students. The names of the teacher candidates in the participants were kept confidential. It is encoded as EMT1, EMT2, ..., EMT66 for elementary mathematics teacher candidate; SET1, SET2, ..., SET58 for special education teacher candidate; PST1, PST2, ..., PST82 for primary school teacher candidate; PT1, PT2, ..., PT58 for preschool teacher candidate.

DATA COLLECTION TOOL

A five-question written opinion test was used to determine teacher candidates ' perceptions of the concept of zero. The first question is "Is it zero a numeral or a number? Explain your answer with its justification."; second question "Is zero a counting number or a natural number? Explain your answer with its justification.". Third, "Is zero an odd or even number? Explain your answer

with its justification.” the question has been asked. In order to describe in detail, the candidate teachers' perceptions of zero, “What comes to your mind when you say zero?" and "Write examples about the use of zero in daily life.” the fourth and fifth questions, such as, were included in the data collection tool. All the selected questions were determined by considering the literature. With these five questions, it was aimed to clearly determine the candidate teachers' understanding of zero. Here, questions that do not require knowledge at the level of expertise were created, as teacher candidates ' perceptions of zero were examined, not to determine their level of knowledge. It was studied by two mathematics educators who specialize in the field of data collection tools. It has been stated by these experts that the questions are appropriate to the research problem. In line with expert opinions, some changes have been made to the question roots. For example, in expert opinion, instead of the “either” conjunct used in the first question, the “or” conjunct was used. In addition, the final version of the data collection tool was given with the pilot implementation with 52 primary school teacher candidates.

Analysis of The Data

To provide in-depth examination due to the case study feature, both content and descriptive analysis of the data was carried out. According to Yıldırım and Şimşek (2013), previously determined categories can be used when analyzing the data in descriptive terms, and the dimensions revealed in the data collection process can also be categories. In the first three questions in the study, categories were created according to the reasoned answers given by the students, and the reasons included in each category were presented in tables. For the last two questions, the data was analyzed using the content analysis method. In content analysis, it is essential to reach relationships that will explain the data. Categories are reached because of this analysis. To do this, data is logically organized, and categories are created according to this arrangement (Yıldırım & Şimşek, 2013). The answers given to fourth question, “What comes to mind when you say zero?”, are categorized into mathematical and semantic expressions. According to the answers written to the fifth question, “Write examples about the use of zero in daily life.”, sample categories with the meaning of absence, with the meaning of the starting point, with the meaning of measurement and containing the meaning of mathematical symbols were created. Direct citations were also made at the necessary points.

Credibility Transferability, Consistency and Confirmability

Lincoln and Guba (1985) stated that different concepts should be used in qualitative studies instead of validity and reliability used in quantitative studies to increase the quality of the studies. These concepts, which are suitable for the nature of qualitative studies, have been determined as credibility, transferability, consistency and confirmability. In order to ensure the credibility of the study, depth-oriented data were collected. In addition, the research process benefited from the opinions of two mathematics educators who are experts in their field. The findings were expressed in rich and detailed tables and figures. Care has been taken to ensure that the categories created are original. These are intended to increase the transferability of research. The consistency between the encodings that the researchers made over different time periods was calculated and the coding reliability percentage was found to be 89%. According to Miles and Huberman (1994), this value should be above 80%. If the coding reliability percentage is more than 80%, it is a condition that expresses the consistency of the research. The data obtained were kept by the researchers to ensure the confirmability of the study.

RESEARCHER'S ROLE

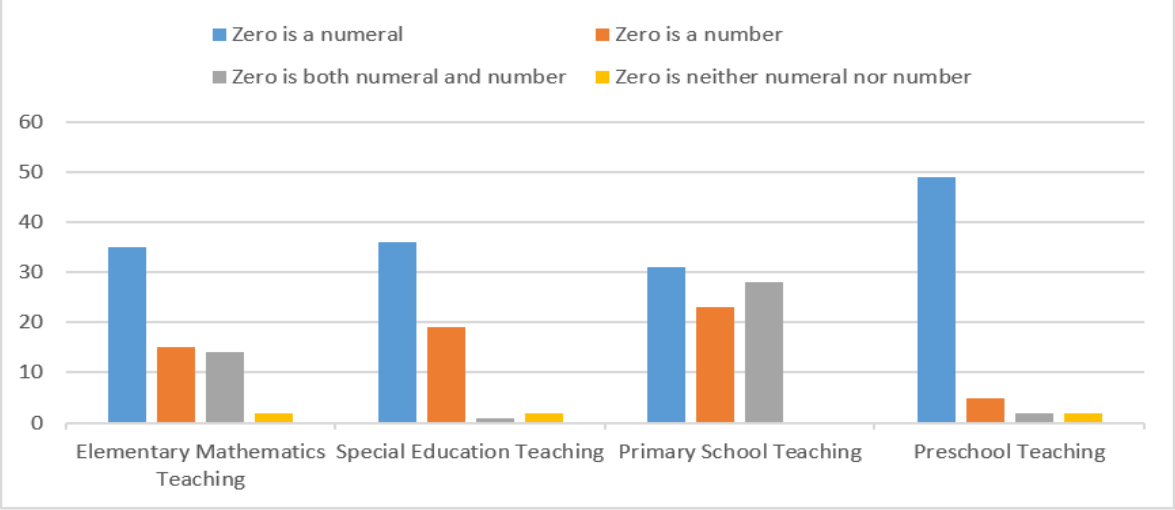
Researchers chose their own lessons and contacted students to be able to gather data from teacher candidates in a comfortable way. Before starting the research application process, teacher candidates were informed about the purpose of the study and an informed consent form was read. The data was collected and analyzed by the researchers themselves. In terms of the ethics of the research, teacher candidates have been promised that the names of teacher candidates will not be used anywhere.

RESULTS

The first question in the data collection tool is "Is it zero a numeral or a number? Explain your answer with its justification." in the form. The frequency distribution chart for the answers the participants wrote to this question according to the sections they were involved in is stated in Figure 1.

Figure 1

Frequency Distribution Graph for the First Question



In Figure 1, it was determined that most teacher candidates (f= 151) expressed zero as a numeral. Almost all preschool teacher candidates (f=49) think of zero as a numeral. Unlike the primary school teacher candidates, those in other departments perceive zero as a numeral, a number, both numeral and number, neither numeral nor number, respectively. Primary school teacher candidates mostly expressed zero after the numeral (f=28) as both numeral and number.

In the first question, the teacher candidates were asked to justify the answers they wrote in detail. To increase the intelligibility of the reasons written by the candidates for the questions, a table was used in the expression of the data. The frequency value of the justifications in each category was calculated. Percentages of calculated frequency values were created in the context of the relevant category. Below is the frequency and percentage table created in this direction (Table 1).

Table 1
Frequency and Percentage Table for Answers to the First Question

Category	Justifications	Departments					
		EMT (f)	SET (f)	PST (f)	PT (f)	Total (f)	%
Zero is a numaral	The numerals are from zero to nine	5	18	14	26	63	42
	Numerals form numbers	19	3	6	8	36	24
	Zero does not mean quantity	6	3	1	6	16	10
	Counting numbers start from one	2	6	5	0	13	9
	No justifications	3	1	4	9	17	11
	Inconsistent description	0	5	1	0	6	4
Zero is a number	Only the numerals correspond to the quantity	2	2	1	0	5	8
	Counting numbers start from zero	1	0	0	0	1	2
	Zero is in the set of integers	1	0	0	0	1	2
	Numerals start from one	7	12	17	3	39	63
	It is used to express absence or uncertainty	4	0	0	0	4	6
	No justifications	0	3	4	1	8	13
	Inconsistent description	0	2	1	1	4	6
Zero is both numeral and number	Numerals form numbers	7	0	9	0	16	36
	Numbers cover numerals	3	0	8	0	11	24
	No justifications	3	1	10	2	16	36
	Inconsistent description	1	0	1	0	2	4
Zero is neither numeral nor number	Zero alone makes no sense	0	1	0	0	1	17
	Inconsistent description	2	1	0	2	5	83

Note. EMT: Elementary mathematics teaching, SET: Special education teaching, PST: Primary school teaching, PT: Preschool teaching, f: The number of teacher candidates in the submitted justifications and %: Percentage of the number of teacher candidates in the submitted justifications according to the relevant category.

Eleven percent of teacher candidates who expressed zero as a numeral when looking at the justifications for the categories contained in Table 1 did not provide a justification, while 4% made inconsistent descriptions. 42% of other teacher candidates who thought of zero as a numeral showed that one of the numerals from zero to nine was also zero as a justification. In contrast to other departments, more than half of the candidates for elementary mathematics teaching alone (f=19) stated that zero is used when writing numbers, and that the numbers are also written with numerals, and therefore they said that zero should also be a numeral. This rationale "Numerals form numbers." presented in the form.

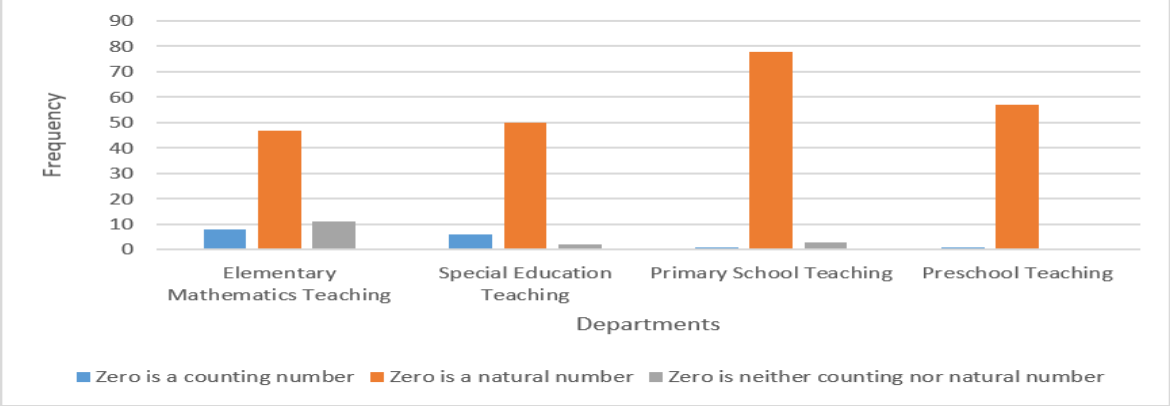
Thirteen percent of teacher candidates who considered zero as a number had no justification; 6% made inconsistent descriptions. 63% of teacher candidates in this category stated that the numerals start from one, so zero can be a number. This expression coded as "The numerals start from one." as a justification is incorrect. It is considered important that this incorrect justification is stated from by the majority.

The number of teacher candidates indicating that zero is both numeral and number is less than the number of teacher candidates in the other categories mentioned above (f=45). Despite this, 36% of these teacher candidates failed to provide justification and 4% made inconsistent statements. This finding shows that the candidate teachers had difficulty in providing a justification for the information they thought was correct intuitively.

"Is zero a counting number or a natural number? Explain your answer with its justification." Figure 2 was created according to the answers given by the candidate teachers to the second question. In Figure 2, the answers given by the teacher candidates are classified according to their departments.

Figure 2

Frequency distribution graph for the second question



In Figure 2, it was determined that most teacher candidates (f=232) considered zero as a natural number. Even looking at Figure 2, it has been concluded that the candidate teachers, albeit in different departments, think of zero as a natural number.

The answers given by the teacher candidates for the second question were evaluated according to their justifications. In Table 2, the justifications for each category were calculated by associating them with the number of teacher candidates in the departments. The number of teacher candidates calculated in the context of each category is presented as a percentage.

When Table 2 is examined, 11% of the candidate teachers stating that zero is a natural number has no justification; 9% of them have inconsistent explanations. It was observed that the least special education teacher candidates (f = 31) tried to provide justification for why zero is a natural number. This indicates a lack in mathematical content knowledge in the department of special education teaching. In the second question, their answer is "Zero is a natural number" 62% of the candidate teachers in the category indicated that counting started at one as a justification. The fact that this justification is based on a given definition shows that some concepts are known by heart by these teacher candidates. Again, 13% of teacher candidates in this category accepted zero as a natural number, citing the lack of an equivalent zero in counting. From here, it is understood that teacher candidates mix the concepts of counting and number.

Candidates of elementary mathematics teaching department (f=5), rather than other departments, accepted that zero is a counting number and offered justifications for this. These justifications "According to the consecutive principle, zero is a counting number", "It means the absence in counting", "Natural numbers start from one". The consecutive principle considered for natural numbers is generalized to counting numbers. Again, the concept of counting and number was found to be mixed. It is an important finding that candidates for elementary mathematics teaching in particular, present natural numbers as a justification that start from one.

Table 2
Frequency and Percentage Table for Answers to the Second Question

Category	Justifications	Departments					
		EMT (f)	SET (f)	PST (f)	PT (f)	Total (f)	%
Zero is a counting number	According to the consecutive principle, zero is a counting number	2	0	0	0	2	13
	It means the absence in counting	2	0	0	0	2	13
	Natural numbers start from one	1	0	0	0	1	6
	No justifications	3	5	1	1	10	62
	Inconsistent description	0	1	0	0	1	6
Zero is a natural number	Counting numbers start from one	25	17	62	39	143	62
	Natural numbers start from zero	0	10	1	0	11	5
	Zero has no equivalent in counting	12	4	8	7	31	13
	No justifications	4	12	4	7	27	11
	Inconsistent description	6	7	3	4	20	9
Zero is neither counting nor natural number	It performs different tasks in different spaces	7	0	0	0	7	44
	None effect	1	0	0	0	1	6
	No justifications	2	2	0	0	4	25
	Inconsistent description	1	0	3	0	4	25

Note. EMT: Elementary mathematics teaching, SET: Special education teaching, PST: Primary school teaching, PT: Preschool teaching, f: The number of teacher candidates in the submitted justifications and %: Percentage of the number of teacher candidates in the submitted justifications according to the relevant category.

Another important situation is that there are candidate mathematics teachers (f = 11) who think that zero is neither counting nor natural number. They did not consider zero as a natural or counting number due to the justifications that zero assumes different tasks in different spaces and has the judgment of being non-existent.

"Is zero an odd or even number? Explain your answer with its justification." the frequency distribution chart created according to the answers written by the candidates to the question is written in Figure 3.

As can be seen in Figure 3, the vast majority of teacher candidates (f=221) think that zero is an even number. Contrary to expectations, some of the teacher candidates in each department (f=43) stated that zero is neither odd nor even.

In Table 3, "Is zero an odd or even number?" the explanations of the candidate teachers who justified their answer to the question were presented.

While thirty-four percent of those who stated that zero is an even number stated the 2n rule as their justification, 19% stated that it is possible to pair with zero and 12% stated that the numbers with the end zero can be divided by two. Although 28% of the candidates stated that zero is an even number, they did not write a justification. Except for the justification that it is accepted that zero is an even number, it is understood that all of the other justifications are consistent.

According to some teacher candidates (f=43), zero is neither odd nor even number. Teacher candidates in this category, 28% presented zero as a neutral and 32% zero as a justification for absence. Given these justifications, the majority of elementary mathematics and special education teacher candidates (f=8) stated the justification for being zero-neutral. According to the majority of primary school and preschool teacher candidates (f=13), the justification why zero is neither

odd nor even is that zero indicates an absence. These teacher candidates took into account the meaning of zero for the use of it in daily life. So they ignored the mathematical meaning of zero.

Figure 3

Frequency Distribution Graph for the Third Question

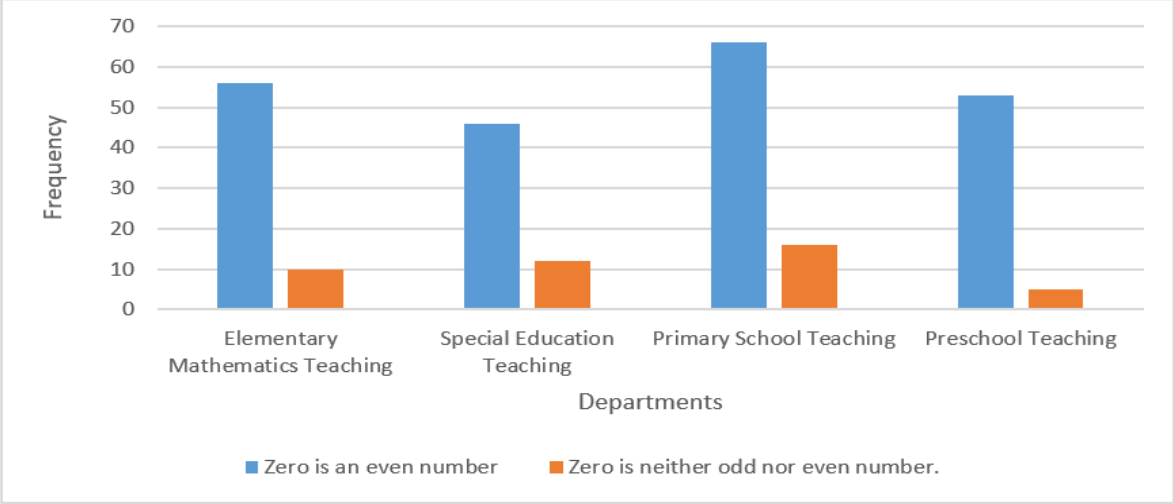


Table 3

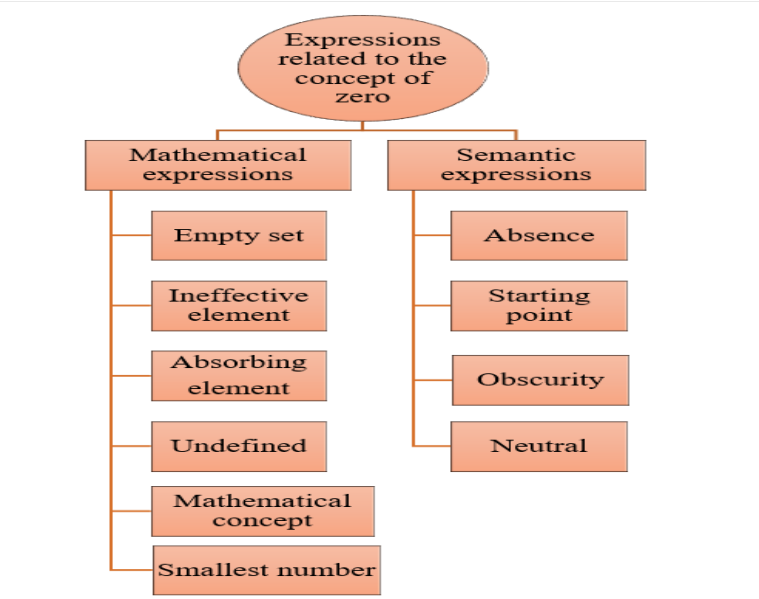
Frequency and Percentage Table for Answers to the Third Question

Category	Justifications	Departments					
		EMT (f)	SET (f)	PST (f)	PT (f)	Total (f)	%
Zero is an even number	It follows the 2n logic	30	14	13	19	76	34
	Numbers with an end of zero can be divided by two	3	9	1	13	26	12
	This is an admission	9	4	2	1	16	7
	Like even numbers, zero indicates equilibrium	1	0	0	0	1	0
	Mapping can be done in pairs with zero	4	4	23	10	41	19
	No justifications	9	15	27	10	61	28
Zero is neither odd nor even	It is an integer	2	0	0	0	2	5
	It is a multiple of both odd and even numbers	2	0	0	0	2	5
	It is neutral	3	5	4	0	12	28
	It is a numeral	1	1	0	0	2	5
	It is a starting point	1	0	0	0	1	2
	Indicates an absence	0	1	9	4	14	32
	No justifications	1	5	3	1	10	23

Note. EMT: Elementary mathematics teaching, SET: Special education teaching, PST: Primary school teaching, PT: Preschool teaching, f: The number of teacher candidates in the submitted justifications and %: Percentage of the number of teacher candidates in the submitted justifications according to the relevant category.

"What comes to mind when you say zero?" the answers written by the candidates to the question were evaluated by the researchers and they were classified as mathematical expressions and semantic expressions and presented in Figure 4.

Figure 4
Teacher Candidates Write About Zero



When the answers to this question were examined, it was found that some teacher candidates (f=12) left this question unanswered. Other teacher candidates gave more than one answer to this question. The answers of the teacher candidates were evaluated according to the number of repetitions of the statements given.

It was determined that teacher candidates used mathematical (f=111) and semantic (f=235) expressions. Mathematical expressions include empty set (f=1), ineffective element (f=53), absorbing element (f=33), undefined (f=1), mathematical concept (f=22) and smallest number (f=1). Among the answers given to this question, the most absorbing and ineffective element expressions (f=88) were used when the candidates were considered as a whole without taking into account the departments. In semantic expressions, absence (f=166), starting point (f=39), obscurity (f=4), neutral (f=26) are among the most commonly written expressions. Almost all of the teacher candidates (166 out of 264 teacher candidates) stated that they considered zero as absence. In other words, the answers given by the teacher candidates to this question are more semantic, and this means that zero indicates absence.

"Write examples about the use of zero in daily life." the samples written by the candidates for the question were evaluated by the researchers and classified as examples with the meaning of absence, samples with the meaning of the starting point, samples with the meaning of measurement, and examples with the meaning of mathematical symbols. These classifications, written in Table 4, are detailed in the sample statement of one teacher candidate in each section.

Table 4.
Examples of teacher candidates responding to the fifth question

Content of the given example	Expressions for related content
Examples with the meaning of absence	*We use it when we talk about absence, not being. For example, if there is no water in the jug, it is like zero liter of water. (EMT3)*
	… For example, a pen is shown to the child. Then, when there is no pen, it can be called a zero pen because it expresses absence. (SET34)
	If a student has no pen, we say he has zero pen. (PST20)
	I am zero on this, zero error, be zero, come up dry, run out of steam, zero tolarance (PT49)
Examples that mean the starting point	*It serves as the starting point for determining the speed of vehicles... (EMT6)*
	...Lets start from zero. (SET9)
	We say zero kilometre for a car that has never been used… (PST39)
	Zero phone, that is, a phone that has never been used. (PT39)
Examples with the meaning of measurement	*Ground floor… As we go upstairs; First floor, second floor, third floor. As we go down from the ground floor, it is called minus the first floor and minus the second floor. (EMT35)*
	... Zero appears in daily life even in clothing. Size zero…(SET1)
	Used in weather conditions. For example, the air temperature of Erzurum today is like zero degrees. (PST39)
	We use zero when we say it' ten o'clock…(PT5)
Examples with mathematical symbol meanings	*We use it when writing numbers …(EMT7)*
	...I use it form IP no, mobile phone no, and some countings…(SET20)
	In blood groups, like 0 Rh. Used to show decimal numbers like, 0,78. In money counting like, 0,75. (PST24)
	For money counting…(PT20)

In Table 4, there are sample expressions of teacher candidates from each department. The daily life examples that the teacher candidates wrote about zero show similarities.

DISCUSSION, CONCLUSION AND RECOMMENDATIONS

In this study, it was aimed to reveal the zero perceptions of teacher candidates. For this purpose, it was determined that the pre-service teachers thought of zero as a numeral and evaluated it as a natural number. As expected, it was also determined that they saw zero as an even number. In addition, it has been concluded that they often use the semantic state of zero instead of the mathematical state. As a result, although it was carried out with teacher candidates studying in different departments, it was found that there were no major differences between the zero understanding of these teacher candidates.

It has been concluded that the teacher candidates for the first question think of zero as a numeral. This result does not match the result of Erdoğan's (2019) study, in which the vast majority of teacher candidates think of zero as a numeral. Öçal and Kızıltaş (2019) revealed that preschool teachers used the first zero symbol when teaching zero. In this study, in particular, the fact that almost all preschool teacher candidates (f=49) think of zero as a numeral parallel the study of Öçal and Kızıltaş (2019). The fact that teacher candidates encountered the concept of numerals before the concept of numbers is considered as one of the reasons for this. Teacher candidates "Numbers from zero to nine are called numerals. So, zero is a numeral." his/her statements support this. Again, it can also be effective for teacher candidates to add the number zero to nine numerals, the

name of the number system used is the decimal system, and the system is said with this name. Venketsamy (2022) stated that although teaching numbers at the preschool level is of great importance, teachers are not experts in this subject. Teachers who focus on the depth of undergraduate mathematics teaching need long-term training in teaching numbers. The reason why pre-school teacher candidates considered zero as just a numeral in this study may be because they did not have a deep mathematics education.

When the results of the first question were examined, it was found that most elementary mathematics teacher candidates (f=19) evaluated the numerals as an alphabet of numbers. In other words, it was determined that elementary mathematics teacher candidates consider zero as a placeholder. When the literature is examined, one of the obstacles to understanding zero is the placeholder property of zero (Anthony & Walshaw, 2004; Sterenberg, 2008; Russell & Chernoff, 2011). Zero, which acts as a placeholder, is considered a symbol used to create numbers. In this study, it is seen that the justifications for elementary mathematics teacher candidates to consider zero as a numeral are supported by the literature. In order to eliminate this situation, the place value must be well understood. Herzog *et al.* (2019) recommends that the development of place value understanding should be developed using the aspects of counting and bundling. Thus, resilient understanding of the decimal place value system will be prevented. It is also understood that teacher candidates who specify zero as a number do not consider the relationship between counting and number. Finally, the justifications for teacher candidates to specify zero as both numeral and number, the concept of a numeral and a number has been interpreted as unknown.

One of the reasons why the number of teacher candidates who see zero as a number is less than the number of teacher candidates who see zero as a numeral is that zero indicates absence. When the literature is examined, it has been revealed that zero cannot be perceived as a number because it expresses absence in daily life (Russell & Chernoff, 2011; Jooste, 2012; Krajcsi *et al.*, 2017; Choonya, 2019). This limited thought makes it difficult to grasp that zero is a number (Wheeler & Feghali, 1983; Anthony & Walshaw, 2004; Levenson *et al.*, 2007). In this context, fewer teacher candidates' evaluation of zero as a number is consistent with the literature. However, teacher candidates showed that zero is not a numeral as a reason for this. This rationale is inconsistent with the relevant studies. It can even be thought that teacher candidates have wrong knowledge.

"Is zero a counting number or a natural number? Explain your answer with its justification." in the second question, it concludes that most teacher candidates (f=232) express zero as a natural number. Because in mathematics textbooks, it is written that a set of natural numbers is created by adding zero to the set of counting numbers. For example Yağcı (2011) expresses the natural numbers as the set formed by counting numbers and zero together in his book. The first set of natural numbers from number sets is described. It is also emphasized that the set of natural numbers starts from zero. For these reasons, it has been concluded that teacher candidates first place the concept of zero in the set of natural numbers.

According to the results obtained from the second question, it was found that many teacher candidates (f=47) who accepted zero as a natural number did not write a justification for this question or made inconsistent statements. According to Sterenberg (2008), evaluating zero as a number means assigning a value to it. In Wheeler and Feghali's (1983) study, teacher candidates did not consider zero as a number. The reason for this is that zero does not have a value, that is, it indicates absence. The difference in the use of zero in the language spoken in daily life and in the language of mathematics is the cause of the occurrence of the above-mentioned situation (Seife, 2000; Anthony & Walshaw, 2004). Jooste (2012) explains that zero is a number as follows: "Zero

is a number such as two or three. Two is the number that represents all sets containing two objects. But zero represents the empty set." given this explanation of Jooste (2012), it is believed that candidate teachers do not have sufficient mathematical knowledge of the concept of zero. It can only be said that the justifications written by elementary mathematics teacher candidates for this question are more conceptual than other departments.

Considering the odd-even state of the objects, the third question was asked to the candidate teachers to get their thoughts on determining whether something is odd or even. When the answers to the third question were examined, it was concluded that zero is an even number according to the candidate teachers. The answers written about why zero is an even number agree with the literature. In literature studies, zero is an even number because it can be divided by 2 (Levenson et al., 2007; Jooste, 2012) and it follows the 2n logic (Levenson *et al.*, 2007; Jooste, 2012; Levenson, 2013; Altoğ, 2016; Erdoğan, 2019).

According to the results obtained from the third question of the data collection tool, it was found that especially the candidates for preschool and primary school teachers considered zero as neither odd nor even number. A similar result was encountered in Toluk Uçar's (2011) study with primary school teacher candidates. In this study, some candidate primary school teachers thought that zero was neither odd nor even. In Jooste's (2012) study, many of the mathematics teachers gave reasons for that zero is neither odd nor even. The reasons for the related study are that integers start from zero, that zero is not included in odd and even numbers, and that zero has no value. In short, the results determined for this question of the research and the justifications written (being a starting point, stating an absence) are very similar to Jooste's (2012) study.

In the fourth question, teacher candidates were asked what zero means to them. It has been concluded that the contents of the written answers are mathematical and semantic. It was determined that teacher candidates, without department differences, used the most semantic expressions (absence, beginning, obscurity, neutral) related to zero. In these semantic expressions, almost all the teacher candidates (f=166) have the result that they indicate absence. Anthony and Walshaw (2004) attribute the reason for this to the fact that children use the expression absence when counting with zero from a very young age. In this study, even teacher candidates perceive zero as absence. It is thought that this makes it difficult for teacher candidates to perceive zero as a number. The result of perceiving zero as absence is like the results of previous studies. Erdoğan (2019) found that some students associate the concept of zero only with absence; Öçal and Kızıltaş (2019) determined that when teaching zero, preschool teachers designed their activities to include the meaning of zero in absence. Sterenberg (2008) states that one of the metaphors of primary school mathematics teachers about zero is absence; Wheeler and Feghali (1983) stated that elementary school teacher candidates perceived zero as absence after symbol; Jooste (2012) stated that many primary school mathematics teachers expressed the value of zero as nonexistence. In a study by Russell and Chernoff (2011), it was stated that one of the two elementary mathematics teachers perceived zero as absence and the other as the starting point. One of the reasons for considering zero as a starting point is that students' knowledge of the zero and infinity is insufficient (Crespo & Nicol, 2006; Nutov, 2021). Nutov (2021), who used visual arts to reveal pre-service mathematics teachers' perceptions of zero and infinity, showed that pre-service teachers took zero as the starting point of infinity. This has been interpreted as an indication that the results of the research are reliable.

Examining the examples written by the teacher candidates about the use of zero in daily life (question five), it was concluded that there are examples that express the meaning of absence, examples that express the meaning of the starting point, examples that express the meaning of

measurement, and examples that contain the meaning of mathematical symbols. Given this situation, it was determined that the ways in which teacher candidates perceive zero coincide with the examples they write from daily life. An understanding that meets only examples that express the meaning of measurement cannot be determined.

In this study, data was collected using a written opinion test. It can be said that the results obtained are limited to this data collection tool. By using more types of data collection tools in future research on or near the subject, how teacher candidates perceive zero can be redefined. This research was conducted with candidates of teachers studying in different departments. Subsequent research can be conducted with teachers who will teach and use the concept of zero. Teacher candidates' use of mathematical language is thought to be a factor affecting their understanding of zero. Evaluating candidate teachers in terms of using mathematical language can help to eliminate the deficiencies in this subject. In addition, in-depth results can be obtained by reducing the number of participants in future studies on this subject. The deficiencies identified in the research can be corrected with additions to undergraduate programs before the service and training studies in-service.

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